

K22P 1605

Reg. No. :

Name :

I Semester M.Sc. Degree (CBSS – Reg./Sup./Imp.) Examination, October 2022
(2019 Admission Onwards)

MATHEMATICS

MAT1C05 : Differential Equations

Time : 3 Hours

Max. Marks : 80

PART – A

Answer **four** questions from this Part. **Each** question carries **4** marks.

1. Locate and classify the singular points of the differential equation :

$$x^3(x - 1)y'' - 2(x - 1)y' + 3xy = 0.$$

2. For the differential equation, $2xy'' + (3 - x)y' - y = 0$. Verify that the origin is a regular singular point.

3. Verify the identity.

$$\sin^{-1} x = xF\left(\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, x^2\right)$$

4. Deduce the relation $\Gamma(n + 1) = n!$.

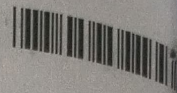
5. Show that $\frac{d}{dx} J_0(x) = -J_1(x)$.

6. If S is defined by the rectangle $|x| \leq a$, $|y| \leq b$, show that the function

$$f(x, y) = xsiny + ycosx, \text{ satisfies the Lipschitz condition.}$$

P.T.O.





PART - B

Answer **four** questions from this Part without omitting **any** Unit. **Each** question carries **16** marks.

Unit - I

7. Find the general solution of $y'' + (x - 3)y' + y = 0$ near $x = 2$.
8. a) Solve by power series method : $y' - y = 0$.
b) Determine whether $x = 0$ is an ordinary point or a regular singular point of the differential equation $2x^2y'' + 7x(x + 1)y' + 3y = 0$.
9. a) Express in the hypergeometric equation
 $(x - A)(x - B)y'' + (C + Dx)y' + Ey = 0$
where $A \neq B$.
b) Find the general solution of the differential equation near the indicated singular point.
 $x(1-x)y'' + \left(\frac{3}{2} - 2x\right)y' + 2y = 0$ at $x = 0$.

Unit - II

10. Derive the Rodrigues Formula for the Legendre equation.
11. Show that

a) $\frac{d}{dx} [x^p J_p(x)] = x^p J_{p-1}(x)$

b) $J_{p+1}(x) = \frac{2p}{x} J_p(x) - J_{p-1}(x)$.

Find the general solution of the following system.

$$\begin{cases} \frac{dx}{dt} = x + y \\ \frac{dy}{dt} = 4x - 2y \end{cases}$$

- b) If the two solutions, $x = x_1(t)$, $y = y_1(t)$ and $x = x_2(t)$, $y = y_2(t)$ of the homogeneous system $\frac{dx}{dt} = a_1(t)x + b_1(t)y$, $\frac{dy}{dt} = a_2(t)x + b_2(t)y$ are linearly independent on $[a, b]$ then prove that $x = c_1x_1(t) + c_2x_2(t)$, $y = c_1y_1(t) + c_2y_2(t)$ is the general solution of the given system on the interval.

Unit – III

3. a) State and prove the Sturm separation theorem.
 b) If $q(x) < 0$, and if $u(x)$ is a non-trivial solution of $u'' + q(x)u = 0$, then show that $u(x)$ has at most one zero.
14. a) Find the exact solution of the initial value problem $y' = 2x(1 + y)$, $y(0) = 0$.
 Starting with $y_0(x) = 0$, calculate $y_1(x)$, $y_2(x)$, $y_3(x)$, $y_4(x)$. (15) 8
- b) Show that $f(x, y) = xy$, satisfies a Lipschitz condition on the rectangle $a \leq x \leq b$ and $c \leq y \leq d$. 8
15. State and prove Picard's theorem. (16)