K22P 1605

Reg. No. : ....

Name : ....

I Semester M.Sc. Degree (CBSS – Reg./Sup./Imp.) Examination, October 2022
(2019 Admission Onwards)
MATHEMATICS
MAT1C05 : Differential Equations

Time : 3 Hours Max. Marks : 80

PART – A

Answer **four** questions from this Part. **Each** question carries **4** marks.

1. Locate and classify the singular points of the differential equation :

   $x^3(x-1)y'' - 2(x-1)y' + 3xy = 0$.

2. For the differential equation, $2xy'' + (3-x)y' - y = 0$. Verify that the origin is a regular singular point.

3. Verify the identity.

   $\sin^{-1} x = xF\left(\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, x^2\right)$

4. Deduce the relation $\Gamma(n+1) = n!$.

5. Show that $\frac{d}{dx} J_0(x) = -J_1(x)$.

6. If S is defined by the rectangle $|x| \le a$, $|y| \le b$, show that the function $f(x, y) = x\sin y + y\cos x$, satisfies the Lipschitz condition.

P.T.O.

PART – B

Answer **four** questions from this Part without omitting **any** Unit. **Each** questio[n] carries **16** marks.

Unit – I

7. Find the general solution of $y'' + (x - 3)y' + y = 0$ near $x = 2$.

8. a) Solve by power series method : $y' - y = 0$.

   b) Determine whether $x = 0$ is an ordinary point or a regular singular point [of] the differential equation $2x^2y'' + 7x(x + 1)y' + 3y = 0$.

9. a) Express in the hypergeometric equation

   $(x - A)(x - B)y'' + (C + Dx)y' + Ey = 0$

   where $A \neq B$.

   b) Find the general solution of the differential equation near the indicated singular point.

   $$x(1-x)y'' + \left(\frac{3}{2} - 2x\right)y' + 2y = 0 \text{ at } x = 0.$$

Unit – II

10. Derive the Rodrigues Formula for the Legendre equation.

11. Show that

   a) $\frac{d}{dx}\left[x^p J_p(x)\right] = x^p J_{p-1}(x)$

   b) $J_{p+1}(x) = \frac{2p}{x} J_p(x) - J_{p-1}(x)$.

Find the general solution of the following system.

$$\begin{cases} \dfrac{dx}{dt} = x + y \\ \dfrac{dy}{dt} = 4x - 2y \end{cases}$$

b) If the two solutions, $x = x_1(t), y = y_1(t)$ and $x = x_2(t), y = y_2(t)$ of the homogeneous system $\dfrac{dx}{dt} = a_1(t)x + b_1(t)y, \dfrac{dy}{dt} = a_2(t)x + b_2(t)y$ are linearly independent on [a, b] then prove that $x = c_1x_1(t) + c_2x_2(t)$, $y = c_1y_1(t)x + c_2y_2(t)$ is the general solution of the given system on the interval.

Unit – III

13. a) State and prove the Sturm separation theorem.

b) If $q(x) < 0$, and if $u(x)$ is a non-trivial solution of $u'' + q(x)u = 0$, then show that $u(x)$ has atmost one zero.

14. a) Find the exact solution of the initial value problem $y' = 2x(1 + y)$, $y(0) = 0$. Starting with $y_0(x) = 0$, calculate $y_1(x), y_2(x), y_3(x), y_4(x)$. (15) 8

b) Show that $f(x, y) = xy$, satisfies a Lipschitz condition on the rectangle $a \le x \le b$ and $c \le y \le d$. 8

15. State and prove Picard's theorem. (16)